

Arranging Chairs

Materials

- Overhead projector, transparency pen
- Interlocking cubes (30 per student)
- Calculators (1 per pair)
- Half-inch graph paper (2–3 sheets per student, 1 transparency) (optional)
- Array Card pages (1 set per student, homework)
- Scissors (1 per student)
- Quart-size resealable plastic bags (1 per student)
- How to Make Array Cards (1 per student, homework)

What Happens

Challenged to find different ways to arrange rows of chairs for an audience, students manipulate 12 cubes to see how many different rectangles they can make. They list the dimensions of these rectangles and the factors of 12. Students work individually and in pairs to determine the factors of other numbers by making rectangles. They also begin making sets of Array Cards for use throughout the investigation. Their work focuses on:

- making rectangles for quantities of 12 and other numbers
- finding factors of 12 and other numbers



Ten-Minute Math: Counting Around the Class Continue to do Counting Around the Class two or three times during this investigation. Remember that this activity is intended to be done outside of math time.

Count by numbers whose patterns are now reasonably familiar to your students: 2's, 5's, 10's, and perhaps 3's, 4's, or 9's. Students can refer to their highlighted charts if they wish.

Ask students to predict ahead. For example, for counting by 3's, ask questions like these:

Who will say 15? Who will say 21? Khanh will be the twelfth student. What number will he say? What number will the student after Khanh say?

Ask questions about how high the counting will go.

Will we reach 50? 100? 200? What do you think will be our final number?

For full instructions and variations on this activity, see p. 91.

Activity

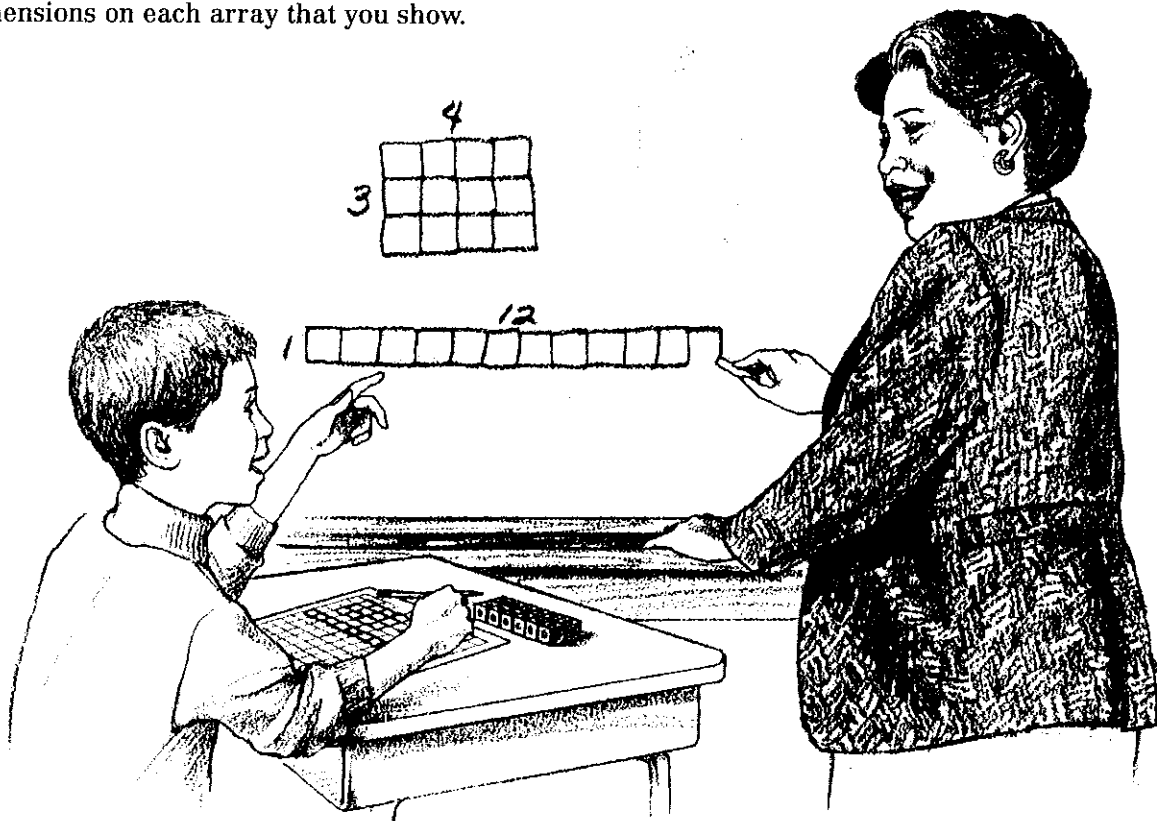
Arranging Chairs in Rectangular Arrays

Introducing Arrays Each student needs 12 cubes to work with. Put 12 cubes on the overhead projector. Briefly explain the task:

Here's a puzzle to solve. We'll call it the Arranging Chairs puzzle. Pretend these 12 cubes are chairs. You want to arrange them in straight rows for an audience to watch a class play. You need to arrange the chairs so that there will be the same number in every row. How many different ways could you do this? How many chairs would be in each row? How many rows would there be? Try many different ways to arrange the chairs, even if some ways seem a bit silly for watching a class play.

❖ **Tip for the Linguistically Diverse Classroom** To support your explanation of the task, model the arranging of four chairs in different ways—one row of 4 across, four rows of 1 (one behind another), and two rows of 2. Make the corresponding arrangement of cubes for each.

Students spend some time making as many different rectangles as they can using the 12 cubes. When they have made several possible arrays, ask them to identify the number of rows and the number of chairs in each row. Show the students' different rectangles by drawing them on an overhead transparency of graph paper, on large graph paper, or on the board. Label the dimensions on each array that you show.



Identify for students the words *array* and *dimension*.

Mathematicians sometimes call things that are grouped this way to form a rectangle an *array*.

Dimension is a name for the length or width of a rectangle. What are the *dimensions* of your rectangles? See how I'm labeling the dimensions of the rectangles as I draw them, the *length* and the *width*.

Use the term *by* when talking about dimensions and students will copy you; for example, "The dimensions of this rectangle are 2 *by* 6." List the pairs of dimensions on the board.

$$3 \times 4 \qquad 2 \times 6 \qquad 1 \times 12$$

$$4 \times 3 \qquad 6 \times 2 \qquad 12 \times 1$$

Have we made all of the possible rectangles? Is our list of dimensions complete? Each of the dimensions on this list is a *factor* of 12. What are all the factors of 12? (1, 2, 3, 4, 6, 12)

Activity

Arranging More Chairs

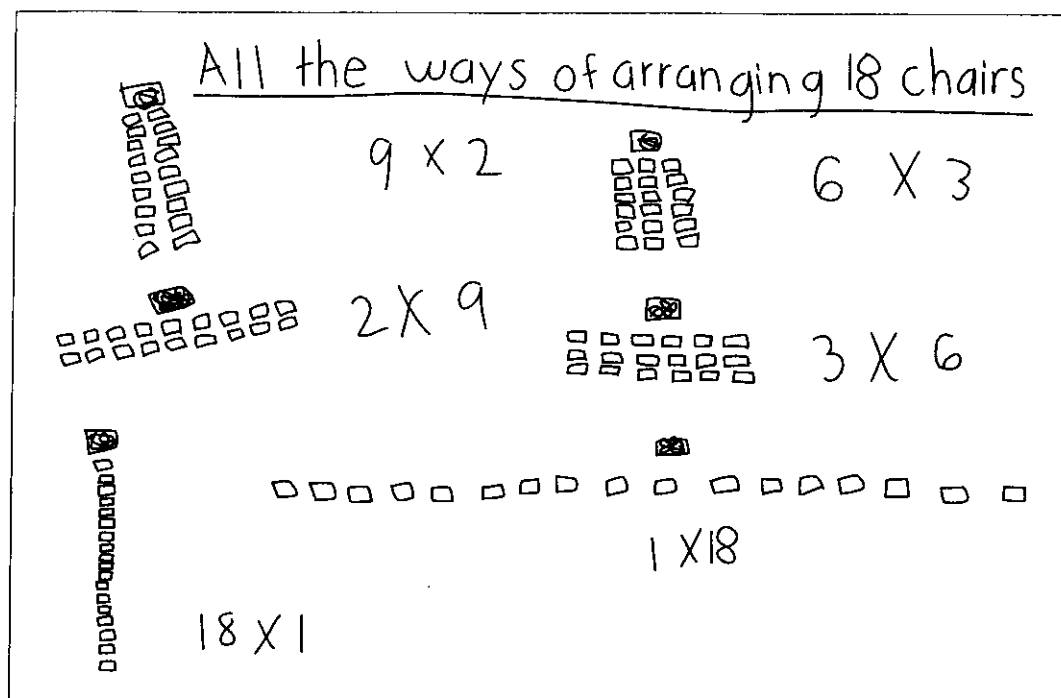
Students continue to work on the Arranging Chairs puzzle, this time with different numbers of chairs. Give each pair of students one of the following numbers to work with:

15 16 18 19 20 21 24 25 30

You might assign the numbers or conduct a drawing. If each pair does more than one number (so that all numbers are done by at least two pairs), different pairs who are working on the same number can compare their answers.

The pairs use cubes to make different arrays of chairs for their number. Then they make drawings of all the arrays they find. If you want, supply half-inch graph paper to make drawing the arrays easier. For each number they work with, students make a list of dimension pairs titled "All the Ways of Arranging ___ Chairs." They may use calculators to find or check the dimensions of their arrays.

See the **Dialogue Box**, Arranging Chairs (p. 47), for an example of a student pair trying to find all possible arrangements of 15 chairs.



When they are finished, invite pairs of students to report their findings, one number at a time. Make a list of the dimensions of the arrays students made for each number. Point out that the number 19 makes only two arrays—1 by 19, and 19 by 1. Remind students about prime numbers—those that didn't turn up on any highlighted charts except their own. Ask:

What other numbers would have only two arrays?

Activity

Making Array Cards

The six pages of Array Cards provide 51 arrays—every possible array representing the multiplication equations in the 2 to 12 tables *with totals up to 50*. If you have purchased the grade 3 manufactured materials for the *Investigations* curriculum, you will have printed sets of these 51 Array Cards that students can use in class. If not, you or an aide will have already made sets of cards for class use. In either case, each student will also benefit from making an individual set of paper Array Cards to use for homework assignments.

Give each student a set of Array Card pages, scissors, and a quart-size resealable plastic bag to hold the cut-apart array cards. Introduce the process of cutting out and labeling the cards as a whole-class activity. Give students time to practice with one or two sheets, and then have them do the rest as homework. Emphasize that all the cards will need to be prepared before the next session. (Some teachers use this student-made set for both homework and classwork. In that case, underscore the importance of preparing and returning the cards to school for the next session.)

Explain the procedure:

1. Start with Array Cards, page 1. The students are to carefully cut out each individual array on the sheet, following the outlines of the grid as exactly as possible. (Seeing the exact outline of each array is important for the array games they will be playing.)
2. Students then label the grid side of each card with the dimensions of the grid.

	2	×	5	
	5	×	2	

3. On the other side of each card (the blank side), students are to write the total number of squares in the grid. They may find it helpful to check the totals with a classmate or with someone at home before they write it permanently. Students may also write one of the dimensions of the grid on the total side, very lightly in pencil, to help them when the arrays are new. These can be erased when the students feel more confident.

2	10	J.P.
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4. Students write their initials on each card (in a corner away from the numbers) and store the labeled cards in the plastic bag.

Before students work independently, you might have them make some of the larger arrays. As students are working, walk around the room and observe whether they understand what to do and how each card should be prepared.

Sessions 1 and 2 Follow-Up

Homework

Cutting Out Array Cards For homework, students finish cutting out the Array Cards that they began making in class. Be sure to send home, besides the Array Card sheets, the plastic storage bags and copies of the sheet How to Make Array Cards as a reminder of how the cards are to be labeled. If this is their classroom set, emphasize that students are to bring their bags of cards back to class with them tomorrow. If this is a set to keep at home, remind students to store their bags of cards in a safe place.